CHAPTER 2 UNIFORMLY ACCELERATED MOTION

1 Graph of uniformly accelerated motion

[Concept]
An object has initial velocity \( u \), accelerates uniformly on a linear track with acceleration \( a \) for a period of time \( t \) until it has a final velocity \( v \) after travelling by a displacement \( s \).

Draw the \( s-t \), \( v-t \) and \( a-t \) graphs of its motion.
(For simplicity, we assume all vector quantities is pointing forward and can be taken as positive)
## 2 Deriving equations for uniformly accelerated motion

### Count number of unknowns and equations

- **5 unknowns:** $u, v, a, s, t$
- **2 equations:**
  1. $s = \text{area of } v-t$
  2. $a = \text{slope of } v-t$

### Equation 1: without $s$

$$a = \frac{v-u}{t} \implies v = u + at$$

### Equation 2: without $a$

$$s = \frac{(u+v)t}{2}$$

### Equation 3: without $v$

Substitute $v = ut + at$ into (Eqn 2)

$$s = ut + \frac{1}{2}at^2$$

### Equation 4: without $t$

$$v^2 - u^2 = 2as$$
A bicycle finishes a 100-meter journey in 9.77 s. Assume that the bicycle starts from rest and moves with uniform acceleration. What is the acceleration of the bicycle throughout the journey?

A. 1.05 m/s²  
B. 2.10 m/s²  
C. 10.2 m/s²  
D. 20.5 m/s²

\[ s = 100 \quad t = 9.77 \quad u = 0 \]

\[ s = ut + \frac{1}{2} at^2 \]

\[ 100 = 0 \cdot 9.77 + \frac{1}{2} a (9.77)^2 \]

\[ a = 2.10 \text{ m/s}^2 \]

A bullet is fired into a wall at 15 m/s. It stops after 0.03 s. Assume that the bullet decelerates uniformly inside the wall.

(a) What is the deceleration of the bullet in the wall? (2 marks)

(b) What is the distance that the bullet penetrates into the wall? (2 marks)

\[ s = ut + \frac{1}{2} at^2 \]

\[ s = 15 \cdot 0.03 + \frac{1}{2} (-500) (0.03)^2 \]

\[ s = 0.225 \text{ m} \]
[Reaction time]

A driver is driving on a straight road with a uniform speed of 9 m s$^{-1}$. When he is 40 m away from a traffic light, the traffic light changes from green to yellow. Suppose the traffic light will change to red in 3 s and the reaction time of the driver is 1 s.

(a) Find the minimum uniform deceleration required if the driver stops in front of the traffic light. (3 marks)

(b) Find the minimum uniform acceleration required if the driver rushes through the traffic light before it turns red. (3 marks)

\[
\begin{align*}
\text{Reaction time} &= 1 \text{ s} \\
\text{\ldots thinking distance} &= 1 \times 9 = 9 \text{ m} \\
\text{\ldots Remaining} &= 31 \text{ m} \\
(a) \quad s &= 31 \text{ m}, \quad u = 9 \text{ m s}^{-1}, \quad v = 0, \quad a = \? \\
& \quad v^2 - u^2 = 2as \\
& \quad 0^2 - 9^2 = 2a(31) \\
& \quad a = -1.31 \text{ m s}^{-2} \\
(b) \quad s &= 31 \text{ m}, \quad u = 9 \text{ m s}^{-1}, \quad t = 2 \text{ s}, \quad a \\
& \quad s = ut + \frac{1}{2}at^2 \\
& \quad 31 = 9(2) + \frac{1}{2}a(2)^2 \\
& \quad a = 6.5 \text{ m s}^{-2}
\end{align*}
\]
A stone is projected vertically upwards from the ground at a speed of 20 m s$^{-1}$. Assume that air resistance is negligible. Take the acceleration due to gravity as 9.81 m s$^{-2}$. Take upward as positive direction.

(a) How high does the stone go? (2 marks)

$$v^2 - u^2 = 2as$$
$$0^2 - 20^2 = 2(-9.81)s$$
$$s = 20.4 \text{ m}$$

(b) Find the time taken for the stone to reach the highest position. (2 marks)

$$v = u + at$$
$$0 = 20 + (-9.81)t$$
$$t = 2.04 \text{ s}$$

(c) What is the state of motion of the stone when it is at the highest position? (1 mark)

momentarily at rest

(d) Find the final velocity of the stone when it returns to the ground. (2 marks)

$$v^2 - u^2 = 2as$$
$$v^2 - 20^2 = 2(-9.81)(0)$$
$$v^2 = 400$$
$$v = 20 \text{ m/s} \text{ or } -20 \text{ m/s} \text{ (downward)}$$
(e) Find the total time of flight.

\[ t = \frac{s}{u + \frac{1}{2}a} \]

\[ s = ut + \frac{1}{2}at^2 \]

\[ 0 = 20t + \frac{1}{2}(-9.81)t^2 \]

\[ 0 = 40t - 9.81t^2 \]

\[ 0 = t(40 - 9.81t) \]

\[ t = 0 \quad \text{or} \quad 40 - 9.81t = 0 \]

\[ t = 4.085 \text{ s (rej.)} \]

(f) Draw the \( s-t \), \( v-t \) and \( a-t \) graphs of the stone.
A diver jumps up from a spring board into the air, reaches the highest point and then falls into the pool. Assume that air resistance is negligible and the acceleration due to gravity is $10 \text{ m s}^{-2}$. Take the upward direction as positive.

(a) The speed of the diver is $8 \text{ m s}^{-1}$ when he leaves the spring board. How long does he take to reach the highest point? (2 marks)

(b) He takes another $1.6 \text{ s}$ before entering the water. Find his speed when entering the water. (1 marks)

(c) What is the height of the spring board above the water? (3 marks)

\[
\begin{align*}
\text{V} & = \text{u} + at \\
& = -16 \text{ m s}^{-1} - 10 \times 2.4 \\
& = -16 \text{ m s}^{-1} \therefore \text{His speed is } 16 \text{ m s}^{-1}.
\end{align*}
\]

\[
\begin{align*}
\text{\textbf{S}} & = ? \\
\text{\textbf{u}} & = +8 \\
\text{\textbf{v}} & = -16 \\
\text{\textbf{a}} & = -10 \\
\text{\textbf{t}} & = 2.4 \text{ s}
\end{align*}
\]

\[
\text{\textbf{S}} = \frac{\text{v}^2 - \text{u}^2}{2 \text{a}} = \frac{(-16)^2 - (8)^2}{2(-10)} = \frac{256 - 64}{-20} = \frac{192}{-20} = -9.6 \text{ m}
\]

\therefore \text{The height is } 9.6 \text{ m}.
A tennis ball is released vertically from a height to the ground. It bounces a few times on the ground before coming to rest. Taking downward as positive, draw the $v-t$ graph of the ball.

1. $\text{slope} = g$
2. $t=0, v=0$
3. $v_+\quad v_-$
4. Sometimes $v=|v_2|$

**Final Remarks**

This chapter focuses on the calculation of linearly accelerated motion. While students are encouraged to memorize the 4 useful equations of motion, you should also aware that you have to keep consistent sign convention in calculation.

In principle, these equations for uniform accelerated motion can also be used in simple case of motion in uniform velocity, by putting $a = 0$. In this case, the equations become $v = u$ and $s = ut$ which are the trivial relations learnt in junior forms.

A special kind of motion encountered is the free fall motion under gravity. Students should know that all objects, without other influences such as air resistance, fall at the same acceleration $9.81\text{ms}^{-2}$ due to gravity on the surface of the Earth.